Useful distributions

univariate normal A random variable $X \in \mathbb{R}$ has a $N(\theta, \sigma^2)$ distribution if $\sigma^2 > 0$ and

$p(x|\theta, \sigma^2) = (2\pi\sigma^2)^{-\frac{1}{2}} e^{-\frac{1}{2\sigma^2}(x-\theta)^2}$ for $-\infty < x < \infty$.

multivariate normal A random vector $X \in \mathbb{R}^p$ has a $MVN(\theta, \Sigma)$ distribution if $\Sigma > 0$ and

$p(x|\theta, \Sigma) = (2\pi)^{-p/2} |\Sigma|^{-1/2} \exp\{-\frac{1}{2}(x-\theta)^T \Sigma^{-1}(x-\theta)\}$

gamma A random variable $X \in (0, \infty)$ has a gamma(a,b) distribution if $a > 0, b > 0$ and

$p(x|a,b) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}$ for $x > 0$.
$E[X|a,b] = a/b$, $Var[X|a,b] = a/b^2$

inverse-gamma A random variable $X \in (0, \infty)$ has an inverse-gamma(a,b) distribution if 1/X has a gamma(a,b) distribution. If X is inverse-gamma(a,b) then the density of X is
$p(x|a,b) = \frac{b^a}{\Gamma(a)} x^{-a-1} e^{-b/x}$ for $x > 0$.
$E[X|a,b] = \frac{b}{a-1}$ if $a >= 1$, ∞ if $0 < a < 1$
$Var[X|a,b] = \frac{b^2}{(a-1)^2(a-2)}$ if $a \geq 2$, ∞ if $0 < a < 2$

inverse-Wishart A random $p \times p$ matrix Σ has an inverse-Wishart distribution if

$p(\Sigma|\nu_0, S_0^{-1}) \propto |\Sigma|^{-(\nu_0+p+1)/2} \times \exp\{-\frac{1}{2} tr(S_0 \Sigma^{-1})\}$.

- the support is $\Sigma > 0$ and Σ symmetric $p \times p$ matrix. $\nu_0 \in \mathbb{N}^+$ and $\nu_0 \geq p$. S_0 is a $p \times p$ symmetric positive definite matrix.
- $E[\Sigma^{-1}] = \nu_0 S_0^{-1}$ and $E[\Sigma] = \frac{1}{\nu_0 - p - 1} S_0$.

binomial A random variable $X \in \{0, 1, \ldots, n\}$ has a binomial(n, θ) distribution if $\theta \in [0,1]$ and

$p(X = x|\theta, n) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$ for $x \in \{0, 1, \ldots, n\}$
$E[X|\theta] = n\theta$, $Var[X|\theta] = n\theta(1-\theta)$

beta A random variable $X \in [0,1]$ has a beta(a,b) distribution if $a > 0, b > 0$ and

$p(x|a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$ for $0 \leq x \leq 1$.
$E[X|a,b] = \frac{a}{a+b}$, $Var[X|a,b] = \frac{ab}{(a+b+1)(a+b)^2}$

Poisson A random variable $X \in \{0, 1, 2, \ldots\}$ has a Poisson(θ) distribution if $\theta > 0$ and
$p(X = x|\theta) = \theta^x \frac{e^{-\theta}}{x!}$ for $x \in \{0, 1, 2, \ldots\}$
$E[X|\theta] = \theta$, $Var[X|\theta] = \theta$

exponential A random variable $X \in [0, \infty)$ has a exponential(θ) distribution if $\theta > 0$ and
$p(x|\theta) = \theta e^{-\theta x}$
$E[X|\theta] = \frac{1}{\theta}$, $Var[X|\theta] = \frac{1}{\theta^2}$